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A study on wave equation and solutions of shallow water on inclined channel

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ABSTRACT: Viscous debris flow in China is well known as lots of intermittent surge flows. These intermittent surge flows are observed not only in China of viscous debris flow but also in the European Alps and other mountains region. But the characteristic of wave motion has not been made clear. It is important to obtain wave equation for wave motion of intermittent surges. We obtain mathematically the wave equation of shallow water with sediment on inclined channel which include intermittent debris flow. Using non-dimensional basic equation as Laplace equation, bottom boundary condition, surface condition (conservation condition of flow surface), and equation of momentum. Using a method of perturbation, Gardner-Morikawa (G-M) transfer, we obtained wave equation of shallow water on inclined channel, and we solved the wave equation on periodical boundary and initial conditions of rectangle and sin function.

1 INTRODUCTION

There are flows of lots of intermittent surges called viscous debris flow in the Jiangjia Gully, Yunnan Province, China (DPRI et al., 1994, 1999). These surges are not the peculiar phenomenon of viscous debris flow and generated not only at the Jiangjia Gully in China but also in the European Alps and other mountains region. Massimo et al. (1997) reported flow characteristic of intermittent debris flow in northern part mountainous district of Italy, Murlimann et al., in Swiss Alps region (2003), Huebl in western part mountainous district of Austria (2010), respectively. It is very important to obtain the wave equation for researching wave motion on these intermittent surge flows. This paper shows the wave equation of shallow water with sediment on inclined channel and solutions of periodical rectangle and sine curve initial condition.

2 BASIC EQUATIONS

To derive the wave equation of shallow water, the assumptions used are in-compressive and non-rotation fluid. Using these assumptions, that is $div\vec{v} = 0$ and $rot\vec{v} = 0$, the potential function can be used. As flow direction is x axis and depth direction y axis, Laplace equation is valid as follow,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1)$$

Defining velocity component of y direction v, a boundary condition of bottom in the flow is

$$v = \frac{\partial \phi}{\partial y} = 0$$
$$\left(y = -h_0, \quad h_0\text{: mean depth}\right) \quad (2)$$

Variance of flow surface from mean depth is defined as η (x, t), conservation condition of fluid of flow surface is satisfied with following equation.

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} \quad (3)$$

The equations of momentum and conservation equation of shallow water with on inclined channel which momentum correction factor is $\beta = 1$ follows,

$$\frac{\partial u}{\partial t} + \beta u \frac{\partial u}{\partial x} + (1-\beta)\frac{u}{A}\frac{\partial A}{\partial t} = g\sin\theta - g\cos\theta\frac{\partial h}{\partial x} - \frac{f'}{2}\frac{u^2}{R} \quad (4)$$

$$\frac{\partial A}{\partial t}+\frac{\partial (Au)}{\partial x}=0 \tag{5}$$

where t: time, u: mean velocity of x compornent, A: cross-sectional flow area, g: acceleration due to gravity, θ: slope angle of the channel, R: hydraulic radius, h: depth of flow, f': friction factor.

Assuming a flow on a rectangle straight channel which width of channel is very large compared with depth, hydraulic radius is $R \cong h$, $\beta = 1$ and friction factor f' is constant by mean depth h_0. From relationships of friction factor $f'/2 = (u_*/u)^2$ and Hazen-Williams coefficient $\varphi = u/u_*$ and research of roll wave of Dressler (1945), third term of right side in equation (4) becomes as follow,

$$\frac{f'}{2}\frac{u^2}{h_0}=\frac{u_*}{\varphi h_0}u \tag{6}$$

Using velocity potential functions, equation (4) becomes as follow.

$$\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 - g\sin\theta x + g\cos\theta h + \frac{u_*}{\varphi h_0}\phi = 0 \tag{7}$$

Using non-dimensional parameters and Gardner-Morikawa (G-M) transfer, basic equations are expressed non-dimensional equations. Non-dimensional parameters with a prime are defined as follows,

$$\phi' = \frac{\phi}{h_0 v_{p0}} \tag{8}$$

$$x' = \frac{x}{h_0} \tag{9}$$

$$y' = \frac{y}{h_0} \tag{10}$$

$$t' = \frac{t v_{p0}}{h_0} \tag{11}$$

$$\eta' = \frac{\eta}{h_0} \tag{12}$$

where v_{p0} is dimensional parameter of velocity and means velocity of coordinate on movement coordinate system. G-M transfer parameters are defined as

$$\xi = \varepsilon^{\frac{1}{2}}\left(x - v_{p0}t\right) \tag{13}$$

$$\tau = \varepsilon^{\frac{3}{2}} t \tag{14}$$

These non-dimensional parameters are expressed as

$$\xi' = \frac{\xi}{h_0} = \varepsilon^{\frac{1}{2}}\left(x' - t'\right) \tag{15}$$

$$\tau' = \frac{\tau v_{p0}}{h_0} = \varepsilon^{\frac{3}{2}} t' \tag{16}$$

From these non-dimensional parameters, basic equations are expressed as follows,

$$\frac{\partial^2 \phi'}{\partial x'^2}+\frac{\partial^2 \phi'}{\partial y'^2}=0 \tag{17}$$

$$\frac{\partial \phi'}{\partial y'}=0, \quad (y'=-1) \tag{18}$$

$$-\frac{\partial \phi'}{\partial y'}+\frac{\partial \eta'}{\partial t'}+\frac{\partial \phi'}{\partial x'}\frac{\partial \eta'}{\partial x'}=0 \quad (y'=0) \tag{19}$$

$$\frac{\partial \phi'}{\partial t'}+\frac{1}{2}\left(\frac{\partial \phi'}{\partial x'}\right)^2 - c_0'^2\tan\theta x' + c_0'^2(1+\eta') + \tan\theta\frac{c_0'^2}{u_0'}\phi' = 0 \tag{20}$$

where,

$$c_0=\sqrt{gh_0\cos\theta}, \quad c_0' = \frac{c_0}{v_{p0}}, \quad u_0' = \frac{u_0}{v_{p0}} \tag{21}$$

3 WAVE EQUATION

Non-dimensional parameter η' and ϕ' are expressed by the perturbative expansion as follows,

$$\eta'(\xi',\tau') = \varepsilon\eta'^{(1)}(\xi',\tau') + \varepsilon^2\eta'^{(2)}(\xi',\tau') + \cdots \tag{22}$$

$$\phi'(\xi',y',\tau') = \varepsilon^{\frac{1}{2}}\left\{\phi'^{(1)}(\xi',y',\tau') + \varepsilon\phi'^{(2)}(\xi',y',\tau') + \cdots\right\} \tag{23}$$

where

$$\eta'^{(1)} = \frac{\eta^{(1)}}{h_0}, \quad \eta'^{(2)} = \frac{\eta^{(2)}}{h_0}, \quad \cdots$$
$$\phi'^{(1)} = \frac{\phi^{(1)}}{h_0 v_{p0}}, \quad \phi'^{(2)} = \frac{\phi^{(2)}}{h_0 v_{p0}}, \quad \cdots \tag{24}$$

and Taylor series expansion by Boussinesq near $y' = 0$ is expressed as follow,

$$\phi'(\xi', y', \tau') = \phi'(\xi', 0, \tau') + \eta' \frac{\partial \phi'(\xi', 0, \tau')}{\partial y'} + \frac{\eta^2}{2}\frac{\partial^2 \phi'(\xi', 0', \tau')}{\partial y'^2} + \cdots \quad (25)$$

Therefore perturbative expansion of non-dimensional equation (20) of momentum is obtained as next equation.

$$\begin{aligned}
&\frac{\partial \phi'}{\partial t'} + \frac{1}{2}\left(\frac{\partial \phi'}{\partial x'}\right)^2 - c_0'^2 \tan\theta x' + c_0'^2(1+\eta') + \tan\theta \frac{c_0'^2}{u_0'}\phi' \\
&= -\varepsilon\left(\frac{\partial \phi'^{(1)}}{\partial \xi'} + \varepsilon \frac{\partial \phi'^{(2)}}{\partial \xi'} + \varepsilon^2 \frac{\partial \phi'^{(3)}}{\partial \xi'} + \cdots\right) \\
&\quad + \varepsilon^2\left(\frac{\partial \phi'^{(1)}}{\partial \tau'} + \varepsilon \frac{\partial \phi'^{(2)}}{\partial \tau'} + \varepsilon^2 \frac{\partial \phi'^{(3)}}{\partial \tau'} + \cdots\right) \\
&\quad + \frac{1}{2}\varepsilon^2\left\{\left(\frac{\partial \phi'^{(1)}}{\partial \xi'}\right)^2 + 2\varepsilon \frac{\partial \phi'^{(1)}}{\partial \xi'}\frac{\partial \phi'^{(2)}}{\partial \xi'} + \varepsilon^2\left(\frac{\partial \phi'^{(2)}}{\partial \xi'}\right) + \cdots\right\} \\
&\quad - c_0'^2 \tan\theta x' + c_0'^2 \\
&\quad + c_0'^2\left(\varepsilon \eta'^{(1)} + \varepsilon^2 \eta'^{(2)} + \varepsilon^3 \eta'^{(3)} + \cdots\right) \\
&\quad + \tan\theta \frac{c_0'^2}{u_0'}\left\{\varepsilon^{\frac{1}{2}}\left(\phi'^{(1)} + \left(\varepsilon \eta'^{(1)} + \varepsilon^2 \eta'^{(2)} + \cdots\right)\frac{\partial \phi'^{(1)}}{\partial y'} + \cdots\right)\right. \\
&\quad + \varepsilon^{\frac{3}{2}}\left(\phi'^{(2)} + \left(\varepsilon \eta'^{(1)} + \varepsilon^2 \eta'^{(2)} + \cdots\right) \times \frac{\partial \phi'^{(2)}}{\partial y'} + \cdots\right) + \varepsilon^{\frac{5}{2}}\left(\phi'^{(3)} + \left(\varepsilon \eta'^{(1)}\right.\right. \\
&\quad \left.\left.\left. + \varepsilon^2 \eta'^{(2)} + \cdots\right)\frac{\partial \phi'^{(3)}}{\partial y'} + \cdots\right)\right\} = 0
\end{aligned} \quad (26)$$

From perturbation expansion of Laplace equation (17), a condition at flow bottom (18), conservation condition of fluid surface (19) and equation (26), the equations classified by ε are as follows,

$O(\varepsilon^0)$ order:

$$\text{from } -c_0'^2 \tan\theta x' + c_0'^2 = 0, \tan\theta = \frac{1}{x'} \quad (27)$$

$O(\varepsilon^{1/2})$order:

$$\frac{\partial^2 \phi'^{(1)}}{\partial y'^2} = 0 \quad (28)$$

$$\frac{\partial \phi'^{(1)}}{\partial y'} = 0, \quad (y' = -1) \quad (29)$$

$$\frac{\partial \phi'^{(1)}}{\partial y'} = 0, \quad (y' = 0) \quad (30)$$

$$\frac{\partial \phi'^{(1)}}{\partial \xi'} + c_0'^2 \eta'(1) + \tan\theta \frac{c_0'^2}{u_0'}\phi'^{(1)} = 0 \quad (31)$$

$O(\varepsilon^{1/2})$ order:

$$\frac{\partial^2 \phi'^{(1)}}{\partial \xi'^2} + \frac{\partial^2 \phi'^{(2)}}{\partial y'^2} = 0 \quad (32)$$

$$\frac{\partial \sigma'^{(2)}}{\partial y'} = 0, \quad (y' = -1) \quad (33)$$

$$-\eta'^{(1)}\frac{\partial^2 \phi'^{(1)}}{\partial y'^2} - \frac{\partial \phi'^{(2)}}{\partial y'} - \frac{\partial \eta'^{(1)}}{\partial \xi'} = 0 \quad (34)$$

$$-\frac{\partial \phi'^{(2)}}{\partial \xi'} + \frac{\partial \phi'^{(1)}}{\partial \tau'} + \frac{1}{2}\left(\frac{\partial \phi'^{(1)}}{\partial \xi'}\right)^2 + c_0'^2 \eta'^{(2)} + \tan\theta \frac{c_0'^2}{u_0'}\left(\eta'^{(1)}\frac{\partial \phi'^{(1)}}{\partial y'} + \phi'^{(2)}\right) = 0 \quad (35)$$

$O(\varepsilon^{5/2})$ order:

$$\frac{\partial^2 \phi'^{(2)}}{\partial \xi^2} + \frac{\partial^2 \phi'^{(3)}}{\partial y'^2} = 0 \quad (36)$$

$$\frac{\partial \phi'^{(3)}}{\partial y'} = 0, \quad (y' = -1) \quad (37)$$

$$-\eta'^{(2)}\frac{\partial^2 \phi'^{(1)}}{\partial y'^2} - \eta'^{(1)}\frac{\partial^2 \phi'^{(2)}}{\partial y'^2} - \frac{\partial \phi'^{(3)}}{\partial y'} - \frac{\partial \eta'^{(2)}}{\partial \xi'} + \frac{\partial \eta'^{(1)}}{\partial \tau'} + \frac{\partial \phi'^{(1)}}{\partial \xi'}\frac{\partial \eta'^{(1)}}{\partial \xi'} = 0, \quad (y' = 0) \quad (38)$$

$$-\frac{\partial \phi'^{(3)}}{\partial \xi'} + \frac{\partial \phi'^{(2)}}{\partial \tau'} + \frac{\partial \phi'^{(1)}}{\partial \xi'}\frac{\partial \phi'^{(2)}}{\partial \xi'} + c_0'^2 \eta'^{(3)} + \tan\theta \frac{c_0'^2}{u_0'}\left(\eta'^{(2)}\frac{\partial \phi'^{(2)}}{\partial y'} + \eta'^{(1)}\frac{\partial \phi'^{(2)}}{\partial y'} + \phi'^{(3)}\right) = 0 \quad (39)$$

Neglecting the term of ϕ' with $u_* \ll u_0$, and from equation (27) to (39), these equations are consolidated in $\eta'^{(1)}$. Then a wave equation is solved as follow. $\eta'^{(1)}$ is substituted for η'.

$$\frac{\partial \eta'}{\partial \tau'} + \frac{1}{2}\left(1 + 2c_0'^2\right)\eta'\frac{\partial \eta'}{\partial \xi'} - \frac{1}{2}\tan\theta\frac{c_0'^2}{u_0'}\frac{\partial^2 \eta'}{\partial \xi'^2} + \frac{1}{2}\left(\frac{1}{c_0'^2} - 1\right)\frac{\partial^3 \eta'}{\partial \xi'^3} = 0 \tag{40}$$

The second term of left side is non-linear which it generates waves of various periods, third is dissipation term which disappear high frequency wave and forth is dispersion term which has a characteristic of a soliton on KdV equation.

A parameter of G-M transfer v_{p0} means that it is the same as the velocity on the movement coordinate system. This parameter v_{p0} is generally used $v_{p0} = \sqrt{g h_0 \cos\theta}$ to obtain a wave equation. In this case, using same expression, therefore it is $v_{p0} = c_0$ and $c_0' = c_0/v_{p0} = c_0/c_0 = 1$. Then equation (40) becomes as follow,

$$\frac{\partial \eta'}{\partial \tau'} + \frac{3}{2}\eta'\frac{\partial \eta'}{\partial \xi'} - \frac{1}{2}\frac{\tan\theta}{u_0'}\frac{\partial^2 \eta'}{\partial \xi'^2} = 0 \tag{41}$$

This equation expresses surface fluctuation from mean depth on the movement coordinate system. From $u_0' = u_0/c_0$, Hazen-Williams coefficient $\varphi = u_0/u_*$ and friction factor $u_0/u_* = \sqrt{2/f''}$, third term of left side of equation (41) is expresses as one of follows.

$$-\frac{1}{2}\frac{\tan\theta}{u_0'} = -\frac{1}{2}\frac{c_0\tan\theta}{u_*\varphi} = -\frac{1}{2}\sqrt{\tan\theta}\sqrt{\frac{f'}{2}} \tag{42}$$

4 ANALYTICAL SOLUTIONS OF WAVE EQUATION

To simplify expression of third term of equation (41), using μ as follow,

$$\mu = \frac{1}{2}\frac{\tan\theta}{u_0'} \tag{43}$$

Equation (43) is substituted for equation (41), it becomes

$$\frac{\partial \eta'}{\partial \tau'} + \frac{3}{2}\eta'\frac{\partial \eta'}{\partial \xi'} = \mu\frac{\partial^2 \eta'}{\partial \xi'^2} \tag{44}$$

This means a Burgers equation. Burgers equation was obtained for wave equation of gas from N-S equation theoretically. In case of Burgers equation, μ means viscosity, but in this case it is a function of channel slope $\tan\theta$ and u_0' based on flow mechanism. And two analytical solutions on different initial conditions are obtained in this paper.

Using $z(\xi', \tau')$ of Cole-Hopf transfer where is

$$\eta' = -\frac{4}{3}\mu\frac{\partial}{\partial \eta'}\ln z = -\frac{4}{3}\mu\frac{1}{z}\frac{\partial z}{\partial \xi'} \tag{45}$$

Non-linear partial differential equation (44) is transformed into below one dimensional linear heat equation by equation (45).

$$\frac{\partial z}{\partial \tau'} = \mu\frac{\partial^2 z}{\partial \xi'^2} \tag{46}$$

In cases of different two initial conditions, the solutions of equation (44) are obtained as following.

4.1 *In case of an initial condition of periodical rectangular wave*

Boundary conditions are periodic condition of period $T = 2l$, and are expressed only from $-l$ to l as follows,

$$\eta' = 0\begin{cases}\xi' = -l \\ \xi' = l\end{cases} \tag{47}$$

An initial condition is a periodical rectangular wave with amplitude a and period $2l$ as follows,

$$\eta'(\xi', 0) = \begin{cases} -\dfrac{a}{2}, & (-l < \xi' < 0) \\ \dfrac{a}{2}, & (0 < \xi' < l) \end{cases} \tag{48}$$

and it is shown in Figure 1.

The boundary and initial conditions on ξ'-η' axis are transferred the conditions on ξ'-z by Cole-Hopf transfer and the solution of equation (46)

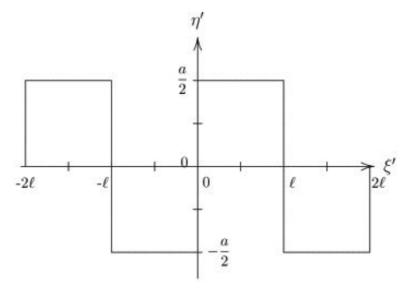

Figure 1. Initial condition.

with these boundary and initial conditions is solved by using Fourier series as follow.

$$z(\xi', \tau') = \frac{1}{lr}\left(e^{lr} - 1\right) + \sum_{n=1}^{\infty} \frac{2lr}{n^2\pi^2 + l^2r^2}\{e^{lr} - \cos(n\pi)\}\cos\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n{}^2\tau'}$$

where

$$r = \frac{3}{\mu}\frac{a}{2}, \quad \lambda_n = \frac{\sqrt{\mu}n\pi}{l} \tag{49}$$

Using Cole-Hopf transfer for equation (49), the solution of equation (44) is obtained as follow.

$$\eta'(\xi', \tau') = \left[a\sum_{n=1}^{\infty} \frac{n\pi}{n^2\pi^2 + l^2r^2}\{e^{lr} - \cos(n\pi)\}\sin\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n{}^2\tau'}\right] \times \left[\frac{1}{lr}\left(e^{lr} - 1\right) + \sum_{n=1}^{\infty} \frac{2lr}{n^2\pi^2 + l^2r^2}\{e^{lr} - \cos(n\pi)\}\cos\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n{}^2\tau'}\right]^{-1} \tag{50}$$

Figure 2 shows an example of numerical results of equation (50). Used parameters are period $l = 1$, amplitude $a = 0.1$ for 1/10 of mean depth h_0, $r = 5.5$ ($\mu = 0.0068$) for slope angle 3 deg., mean depth 1.0 cm and mean velocity $u_0 = 120$ cm/s, and non-dimentional time $\tau' = 0.2, 2.5, 5$ (line color blue, red, green respectively).

The results show that shape of wave changes from rectangle to round with the progress of time and peak of wave shifts in a forward, and a gradient of shape from peak to bottom becomes steep. Figure 3 shows another calculated results of equation (50) where $a = 1$, $\tau' = 2$ and $r = 5, 20, 40$ (line color blue, red green respectively). Increase of r means decrease of slope angle or increase of mean velocity for $r = (3/4\mu)(a/2)$ and $\mu = (1/2)(\tan\theta/u_0{}')$. These calculated results show that the peak of wave becomes higher with increase of r and the gradient from peak to bottom becomes steeper too.

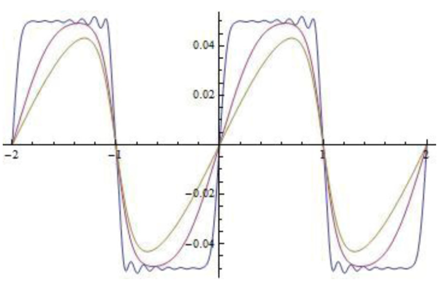

Figure 2. Example of calculated results.

4.2 *In case of an initial condition of periodical sine wave*

Boundary condition is periodic condition of period $T = 2l$, and is expressed only from $-l$ to l as follows,

$$\eta' = 0\begin{cases}\xi' = -l \\ \xi' = l\end{cases} \tag{51}$$

Initial condition is a periodical sine wave with amplitude and period $2l$ as follows,

$$\eta'(\xi', 0) = \begin{cases} -\dfrac{a}{2}\sin\left(\dfrac{\pi\xi'}{l}\right), & (-l < \xi' < 0) \\ \dfrac{a}{2}\sin\left(\dfrac{\pi\xi'}{l}\right), & (0 < \xi' < l) \end{cases} \tag{52}$$

and Figure 4 shows the initial condition.

As same as a case of initial condition of periodic rectangular wave, the boundary and initial conditions on $\xi - \eta$ axis are transferred the conditions on

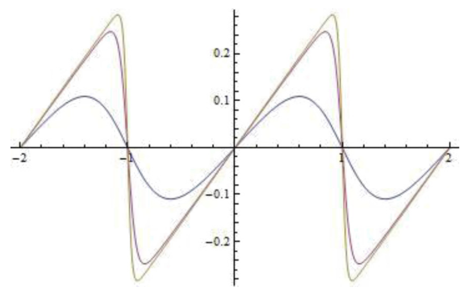

Figure 3. Example of calculated results.

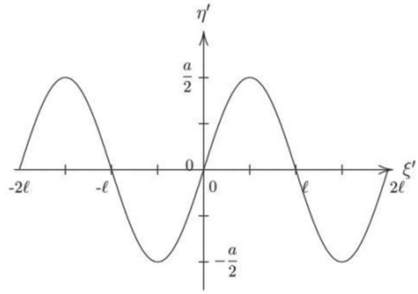

Figure 4. Initial condition of sine wave.

ξ-z by Cole-Hopf transfer and the solution of equation (46) is solved by using Fourier series as follow.

$$z(\xi',\tau') = e^{\frac{lr}{\pi}} J_0\left(\frac{lr}{\pi}\right) + \sum_{n=1}^{\infty}\left\{\frac{1}{l}\int_{-l}^{i} Exp\left[\frac{lr}{\pi}\cos\frac{\pi v}{l} + \frac{lr}{\pi}\right] \times \cos\frac{n\pi v}{l}dv\right\}\cos\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n^2\tau'}$$

where

$$J_0 : \text{the first kind Bessel function, } \lambda_n = \frac{\sqrt{\mu}n\pi}{l} \tag{53}$$

And equation (53) is substituted for equation (45), a solution η' of equation (44) is obtained as follows.

$$\eta'(\xi',\tau') = \left[\frac{a}{2r}\sum_{n=1}^{\infty}\left\{\frac{n\pi}{l^2}\int_{-l}^{l} Exp\left[\frac{lr}{\pi}\cos\frac{\pi v}{l} + \frac{lr}{\pi}\right]\cos\frac{n\pi v}{l}dv\,\sin\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n^2\tau'}\right\}\right] \times \left[e^{\frac{lr}{\pi}} J_0\left(\frac{lr}{\pi}\right) + \sum_{n=1}^{\infty}\left\{\frac{1}{l}\int_{-l}^{l} Exp\left[\frac{lr}{\pi}\cos\frac{\pi v}{l} + \frac{lr}{\pi}\right]\cos\frac{n\pi v}{l}dv\cos\left(\frac{n\pi\xi'}{l}\right)e^{-\lambda_n^2\tau'}\right\}\right]^{-1} \tag{54}$$

Using parameters of $l = 1$, $a = 1$, $r = 5.5$ and $\tau' = 0.2, 2.5, 5$ (line color blue, red, green respectively), calculated results are shown in Figure 5 which the peak of wave decrease and shift in a forward with the progress of time, and a gradient of shape from peak to bottom becomes steep as same as rectangular initial condition.

Another calculated results of equation (50) with $a = 1$, $\tau' = 2$ and $r = 5$, 20, 40 (line color blue, red green respectively) are shown in Figure 6. When r increases, the peak of wave becomes higher. This figure is almost same as figure 3. This means that

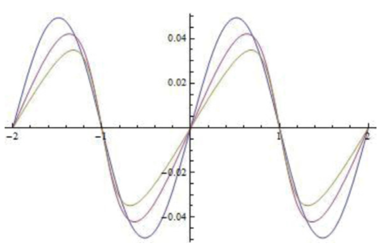

Figure 5. Calculated results ($\tau' = 0.2, 2.5, 5$).

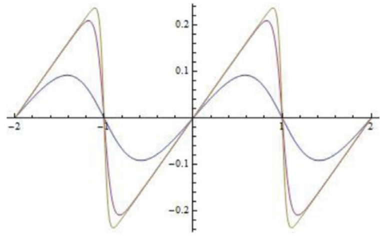

Figure 6. Calculated results ($r = 5.5, 20, 40$).

in spite of different initial condition, shape of wave becomes similar or almost same with progress time.

5 CONCLUSIONS

Using equation of momentum of shallow water on inclined channel, the wave equation (40) was obtained. In case of $v_{p0} = \sqrt{gh_0\cos\theta}$ on G-M transfer, it is expressed as the wave equation (41). The equation (41) was solved on two different periodic initial conditions of rectangular and sine curve. These solutions show similar or almost same shape of wave with progressive time in spite of different initial conditions.

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